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Heavy Quark production at the TEVATRON and HERA using k_t - factorization with CCFM evolution

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Abstract: The application of k_t-factorization supplemented with the CCFM small-x evolution equation to heavy quark production at the TEVATRON and at HERA is discussed. The $b\bar{b}$ production cross sections at the TEVATRON can be consistently described using the k_t-factorization formalism together with the unintegrated gluon density obtained within the CCFM evolution approach from a fit to HERA F_2 data. Special attention is drawn to the comparison with measured visible cross sections, which are compared to the hadron level Monte Carlo generator CASCADE.

1 Introduction

The calculation of inclusive quantities, like the structure function $F_2(x,Q^2)$ at HERA, performed in NLO QCD is in perfect agreement with the measurements. However, Catani argues, that the NLO approach, although phenomenologically successful for $F_2(x,Q^2)$, is not fully satisfactory from a theoretical viewpoint, because *"the truncation of the splitting functions at a fixed perturbative order is equivalent to assuming that the dominant dynamical mechanism leading to scaling violations is the evolution of parton cascades with strongly ordered transverse momenta"* [1]. As soon as exclusive quantities like jet or heavy quark production are investigated, the agreement between NLO coefficient functions convoluted with NLO DGLAP [2–5] parton densities and the data is not at all satisfactory: large so-called K-factors (normalization factors) [6–9] are needed to bring the NLO calculations close to the data ($K \sim 2-4$ for bottom production at the TEVATRON), indicating that in the calculations a significant part of the cross section is still missing.

At small x the structure function $F_2(x,Q^2)$ is proportional to the sea quark density, and the sea-quarks are driven via the DGLAP evolution equations by the gluon density. Standard QCD fits determine the parameters of the initial parton distributions at a starting scale Q_0. With help of the DGLAP evolution equations these parton distributions are then evolved to any other scale Q^2, with the splitting functions still truncated at fixed $O(\alpha_s)$ (LO) or $O(\alpha_s^2)$ (NLO). Any physics process in the fixed order scheme is then calculated via collinear factorization into the coefficient functions $C^a(\frac{x}{z})$ and collinear (independent of k_t) parton density functions: $f_a(z,Q^2)$:

$$\sigma = \sigma_0 \int \frac{dz}{z} C^a(\frac{x}{z}) f_a(z,Q^2) \quad (1)$$

At large energies (small x) the evolution of parton densities proceeds over a large region in rapidity $\Delta y \sim \log(1/x)$ and effects of finite transverse momenta of the partons may become increasingly important. Cross sections can then be k_t - factorized [10] into an off-shell (k_t dependent) partonic cross section $\hat{\sigma}(\frac{x}{z}, k_t)$ and a k_t - unintegrated parton density function $\mathcal{F}(z, k_t)$:

$$\sigma = \int \frac{dz}{z} d^2 k_t \hat{\sigma}(\frac{x}{z}, k_t) \mathcal{F}(z, k_t) \tag{2}$$

The unintegrated gluon density $\mathcal{F}(z, k_t)$ is described by the BFKL [11–13] evolution equation in the region of asymptotically large energies (small x). An appropriate description valid for both small and large x is given by the CCFM evolution equation [14–17], resulting in an unintegrated gluon density $\mathcal{A}(x, k_t, \bar{q})$, which is a function also of the additional evolution scale $\bar{q}$ described below.

In [18,1] Catani argues that by explicitly carrying out the k_t integration in eq.(2) one can obtain a form fully consistent with collinear factorization: the coefficient functions and also the DGLAP splitting functions leading to $f_a(z, Q^2)$ are no longer evaluated in fixed order perturbation theory but supplemented with the all-order resummation of the $\alpha_s \log 1/x$ contribution at small x.

In this paper bottom production at the TEVATRON and at HERA is investigated using the k_t - factorization approach. The unintegrated gluon density has been obtained previously in [19] from a CCFM fit to the HERA structure function $F_2(x, Q^2)$. All free parameters are thus fixed and absolute predictions for bottom production can be made. The universality of the unintegrated CCFM gluon distribution will be tested by the application to TEVATRON and HERA results.

The basic features of the CCFM evolution equation are recalled and the unintegrated gluon density is investigated. Then the calculations for $b\bar{b}$ production at the TEVATRON is presented as well as calculations of the visible cross section for $b\bar{b}$ production at HERA.

2 The CCFM evolution equation

A solution of the CCFM evolution equation, which properly describes the inclusive structure function $F_2(x, Q^2)$ and also typical small x final state processes at HERA has been presented in detail in [19]. Figure 1 illustrates the pattern of QCD initial-state radiation in a small-x event at a $p\bar{p}$ collider, together with labels for the kinematics. According to the CCFM evolution equation, the emission of partons during the initial cascade is only allowed in an angular-ordered region of phase space. The maximum allowed angle Ξ is defined by the hard scattering quark box, producing the heavy quark pair. In terms of Sudakov variables the quark pair momentum is written as:

$$p_q + p_{\bar{q}} = \Upsilon (p^{(1)} + \Xi p^{(2)}) + Q_t \tag{3}$$

where $p^{(1)}$ and $p^{(2)}$ are the four-vectors of incoming protons, respectively and Q_t is the transverse momentum of the quark pair in the laboratory frame. Similarly, the momenta p_i of the gluons emitted during the initial state cascade are given by (here treated massless):

$$p_i = \upsilon_i (p^{(1)} + \xi_i p^{(2)}) + p_{ti} \ , \quad \xi_i = \frac{p_{ti}^2}{s \upsilon_i^2}, \tag{4}$$

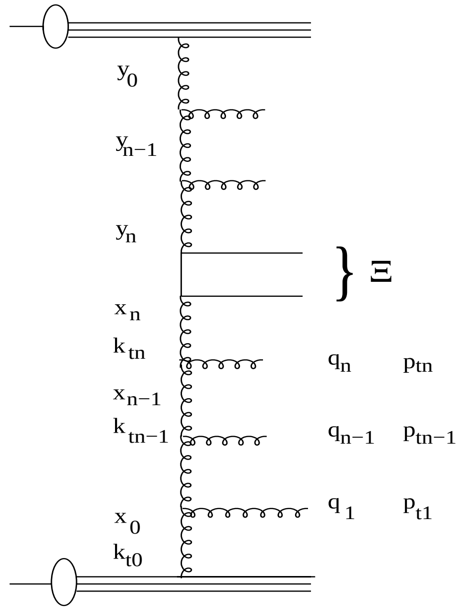

Figure 1: *Kinematic variables for multi-gluon emission. The t-channel gluon four-vectors are given by k_i and the gluons emitted in the initial state cascade have four-vectors p_i. The minimum (maximum) angle for any emission is obtained from the quark box, as indicated with Ξ.*

with $\upsilon_i = (1-z_i)x_{i-1}$, $x_i = z_i x_{i-1}$ and $s = (p^{(1)} + p^{(2)})^2$ being the squared center of mass energy. The variable ξ_i is connected to the angle of the emitted gluon with respect to the incoming proton and x_i and υ_i are the momentum fractions of the exchanged and emitted gluons, while z_i is the momentum fraction in the branching $(i-1) \to i$ and p_{ti} is the transverse momentum of the emitted gluon i.

The angular-ordered region is then specified by (for the lower part of the cascade in Fig. 1, the upper part is obtained by properly exchanging the variables):

$$\xi_0 < \xi_1 < \cdots < \xi_n < \Xi \tag{5}$$

which becomes:

$$z_{i-1} q_{i-1} < q_i \tag{6}$$

where the rescaled transverse momenta q_i of the emitted gluons is defined by:

$$q_i = x_{i-1}\sqrt{s\xi_i} = \frac{p_{ti}}{1-z_i} \tag{7}$$

The CCFM equation for the unintegrated gluon density can be written [17,19–21] as an integral equation:

$$\mathcal{A}(x, k_t, \bar{q}) = \mathcal{A}_0(x, k_t, \bar{q}) + \int \frac{dz}{z} \int \frac{d^2 q}{\pi q^2} \Theta(\bar{q} - zq) \Delta_s(\bar{q}, zq) \tilde{P}(z, q, k_t) \mathcal{A}\left(\frac{x}{z}, k_t', q\right) \tag{8}$$

with $\vec{k}_t' = |\vec{k}_t + (1-z)\vec{q}|$ and $\bar{q}$ being the upper scale for the last angle of the emission: $\bar{q} > z_n q_n$, $q_n > z_{n-1} q_{n-1}$, ..., $q_1 > Q_0$. Here q is used as a shorthand notation for the 2-dimensional vector of the rescaled transverse momentum $\vec{q} \equiv \vec{q}_t = \vec{p}_t/(1-z)$. The splitting function $\tilde{P}(z, q, k_t)$ and the Sudakov form factor $\Delta_s(\bar{q}, zq)$ are given explicitly in [19].

2.1 The unintegrated gluon density

In [19] the unintegrated gluon density $x\mathcal{A}(x, k_t^2, \bar{q})$ has been obtained from a fit to the structure function $F_2(x, Q^2)$ [1].

In Fig. 2 the CCFM unintegrated gluon density distribution as a function of x and k_t^2 is shown and compared to

$$\mathcal{F}(x, k_t^2) \simeq \left. \frac{dxG(x, \mu^2)}{d\mu^2} \right|_{\mu^2 = k_t^2} \tag{9}$$

with $xG(x, \mu)$ being the collinear gluon density of GRV 98 [23] in LO and NLO.

The unintegrated gluon density can be related to the integrated one by:

$$xG(x, \mu)|_{\mu=\bar{q}} \simeq \int_0^{\bar{q}^2} dk_t^2 x\mathcal{A}(x, k_t^2, \bar{q}) \tag{10}$$

Here the dependence on the scale of the maximum angle $\bar{q}$ is made explicit: the evolution proceeds up to a maximum angle related to $\bar{q}$, which plays the role of the evolution scale in the collinear parton densities. This becomes obvious since

$$\bar{q}^2 = x_g^{(2)} \Xi s = x_g^{(1)} x_g^{(2)} s = \hat{s} + Q_t^2 \tag{11}$$

The last expression is derived by using $p_Q + p_{\bar{Q}} \simeq x_g^{(2)} p_2 + x_g^{(1)} p_1 + Q_t$, $\Xi \simeq x_g^{(1)} / x_g^{(2)}$ and and $\hat{s} = x_g^{(1)} x_g^{(2)} s - Q_t^2$. This can be compared to a possible choice of the renormalization and factorization scale μ^2 in the collinear approach with $\mu^2 = Q_t^2 + 4 \cdot m_Q^2$ and the similarity between μ and $\bar{q}$ becomes obvious.

In Fig. 3 the CCFM gluon density integrated over k_t according to eq.(10) is compared to the gluon densities of GRV 98 [23] in LO and NLO. It is interesting to note that the CCFM gluon density is flat for $x \to 0$ at the input scale $Q = 1$ GeV. Even at larger scales the collinear gluon densities rise faster with decreasing x than the CCFM gluon density. However, after evolution and convolution with the off-shell matrix element the scaling violations of $F_2(x, Q^2)$ and the rise of F_2 towards small x is reproduced, as shown in [19, Fig. 4 therein]. A similar trend is observed in the collinear fixed order calculations, when going from LO to NLO: at NLO the gluon density is less steep at small x, because part of the x dependence is already included in the NLO P_{qg} splitting function, as argued in [1].

From Fig. 3 one can see, that the integrated gluon density from CCFM is larger in the medium x range, than the ones from the collinear approach. Due to an additional $1/k_t^2$ suppression in the off-shell matrix elements, the gluon density obviously has to be larger to still reproduce the same cross section. In addition only gluon ladders are considered in the k_t - factorization approach used here, which means that the sea quark contribution to the structure function $F_2(x, Q^2)$ comes entirely from boson gluon fusion, without any contribution from the intrinsic quark sea. One also should remember, that the relation in eq.(10) is only approximately true, since the gluon density itself is not a physical observable.

[1] A Fortran program for the unintegrated gluon density $x\mathcal{A}(x, k_t^2, \bar{q})$ can be obtained from [22]

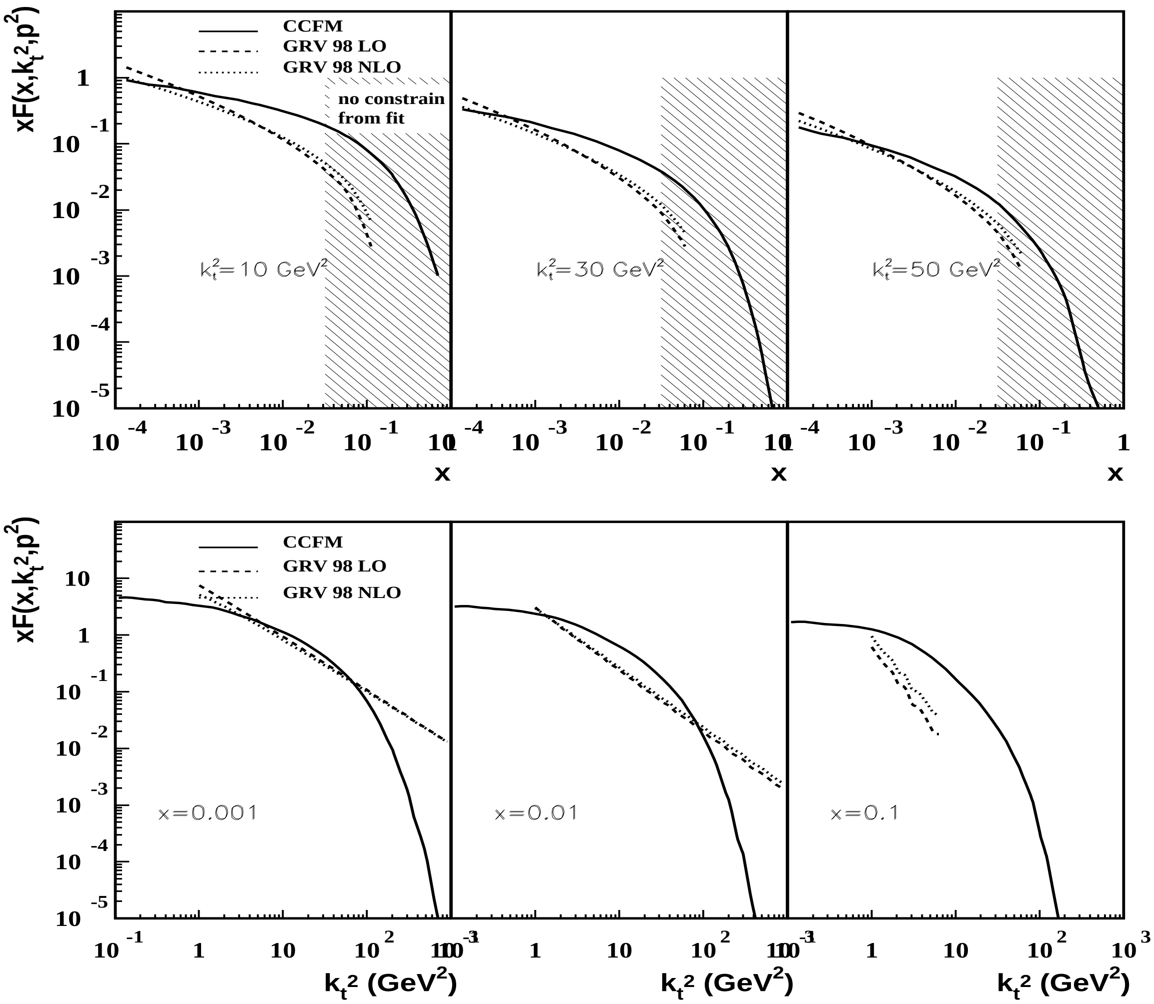

Figure 2: *The CCFM k_t dependent (unintegrated) gluon density [19,22] at $\bar{q} = 10$ GeV as a function of x for different values of k_t^2 (upper) and as a function of k_t^2 for different values of x (lower) compared to $\frac{dxG(x,\mu^2)}{d\mu^2}$ with $xG(x,\mu)$ being the collinear gluon density of GRV 98 [23] in LO and NLO.*

3 $b\bar{b}$ production at the TEVATRON

The cross section for $b\bar{b}$ production in $p\bar{p}$ collision at $\sqrt{s} = 1800$ GeV is calculated with Cascade [19,22], which is a Monte Carlo implementation of the CCFM approach described above. The off-shell matrix element as given in [10] for heavy quarks is used with $m_b = 4.75$ GeV. The scale μ used in $\alpha_s(\mu^2)$ is set to $\mu^2 = m_T^2 = m_b^2 + p_T^2$ (as in [19]), with p_T being the transverse momentum of the heavy quarks in the $p\bar{p}$ center-of-mass frame. In Fig. 4 the prediction for the cross section for $b\bar{b}$ production with pseudo-rapidity $|y^b| < 1$ is shown as a function of p_T^{min} and compared to the measurement of D0 [7]. Also shown is the NLO prediction from [24, taken from [7]]. In Fig. 5 the measured cross section of CDF [6] is shown for 3 values of $p_T^{min}(\bar{b})$ with the kinematic constraint of $|y^b|, |y^{\bar{b}}| < 1$ and $p_T^{min}(b) > 6.5$ GeV together with the

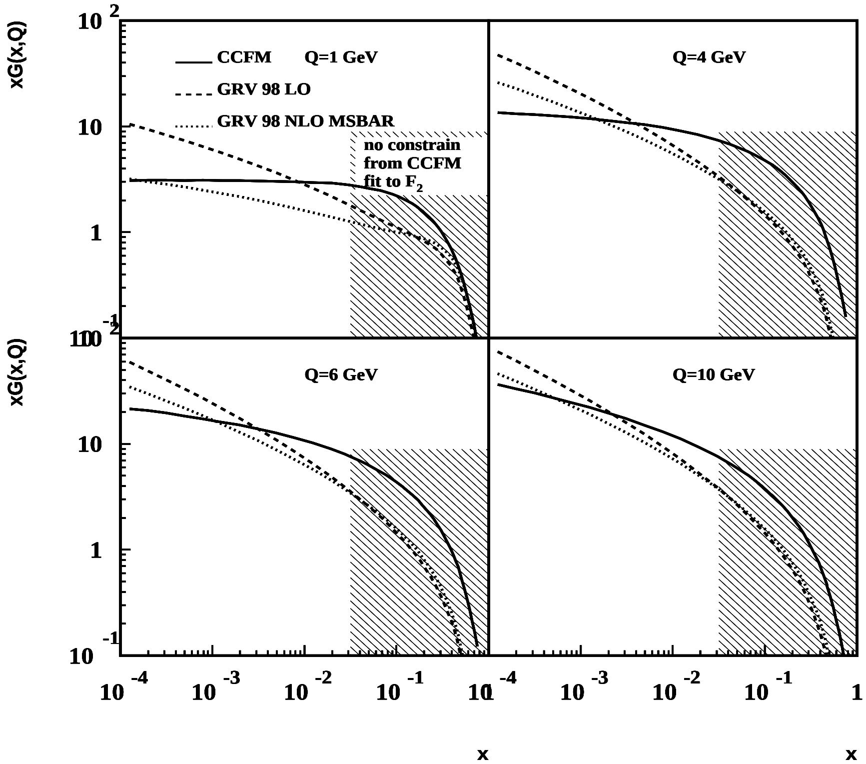

Figure 3: *The CCFM gluon density (solid line) integrated over k_t as a function of x for different values of Q. For comparison the GRV 98 [23] gluon density in LO (dashed line) and NLO (dotted line) is also shown.*

prediction from CASCADE and the NLO calculation from [25, taken from [6]]. In all cases the NLO calculation used $m_b = 4.75$ GeV and the factorization and renormalization scales were set to $\mu^2 = m_T^2 = m_b^2 + p_T^2$. Both, D0 and CDF measurements are above the NLO predictions by a factor of ~ 2. The CASCADE predictions are in reasonable agreement with the measurements. A similarly good description of the D0 and CDF data has been obtained in [27] using also k_t -factorization but supplemented with a BFKL type unintegrated gluon density. It is interesting to note, that the CCFM unintegrated gluon density has been obtained from inclusive $F_2(x, Q^2)$ at HERA. In this sense, the prediction of CASCADE is a parameter free prediction of the $b\bar{b}$ cross section in $p\bar{p}$ collisions. This also shows for the first time evidence for the universality of the unintegrated CCFM gluon distribution.

Since CASCADE generates full hadron level events, direct comparisons with measured cross sections for the production of b quarks decaying semi-leptonically into muons are possible. The muon cross sections as a function of the transverse momentum p_T^μ and pseudo-rapidity $|y^\mu|$ as measured by D0 [26] are compared to the CASCADE prediction in Fig. 6 and Fig. 7. Both the p_T^μ and $|y^\mu|$ cross sections are well described, whereas NLO calculations underestimate the cross sections by a factor of ~ 4 in the small angle range [26].

It has been argued in [28], that within the collinear approach supplemented with DGLAP

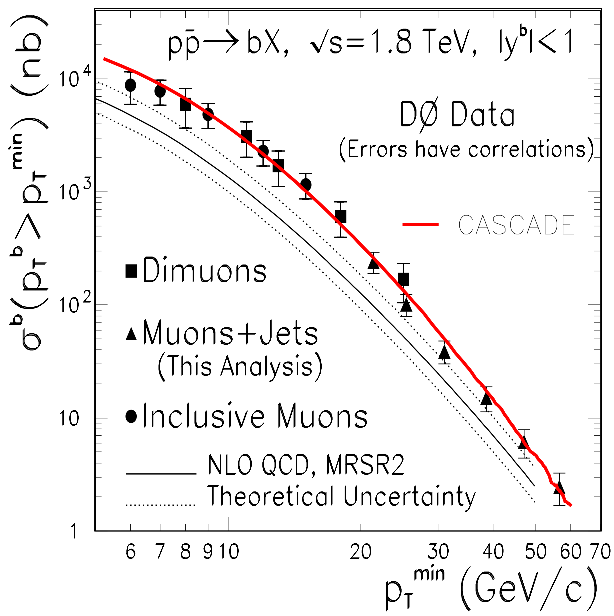

Figure 4: *Cross section for $b\bar{b}$ production with $|y^b| < 1$ as a function of p_T^{min}. Shown are the D0 [7] data points, the fixed order NLO prediction, and the prediction of* CASCADE.

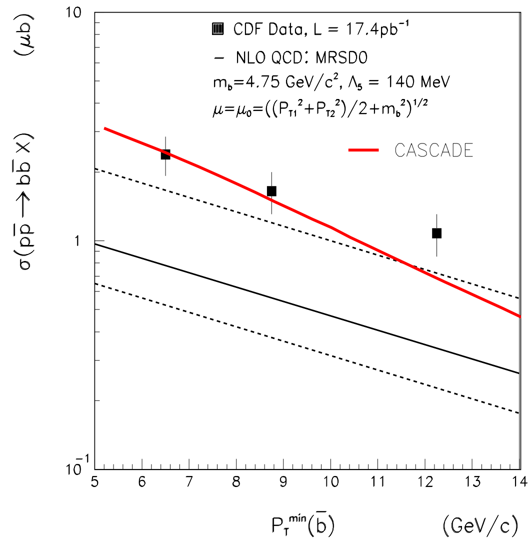

Figure 5: *Cross section for $b\bar{b}$ production with $|y^b| < 1$ and $p_T^{min}(b) > 6.5$ GeV as a function of $p_T^{min}(\bar{b})$. Shown are the CDF [6] data points, the fixed order NLO prediction, and the prediction of* CASCADE.

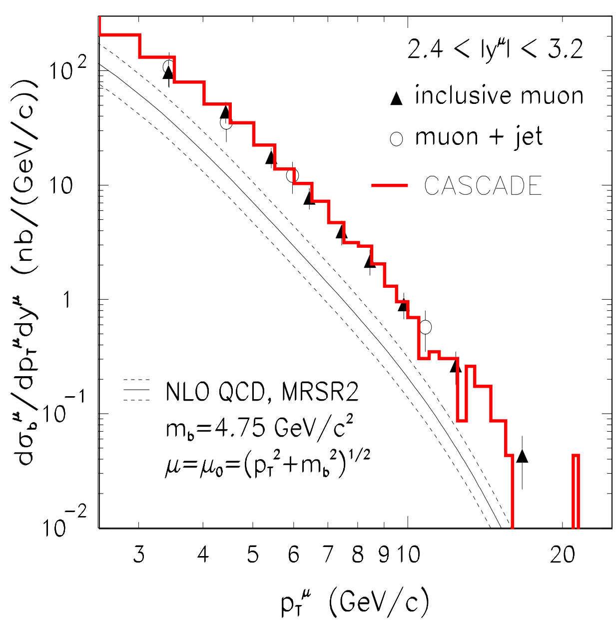

Figure 6: *Cross section for muons from b-quark decays as a function of p_T^μ (per unit rapidity) as measured by D0 [26] compared to the prediction of* CASCADE *.*

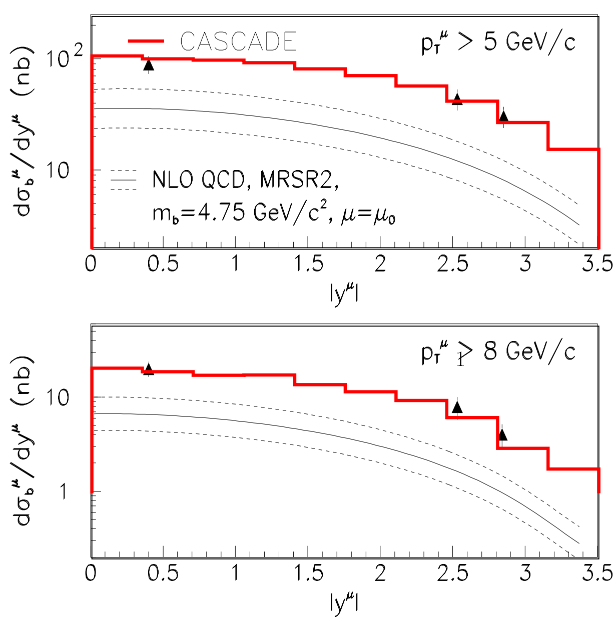

Figure 7: *Cross section for muons from b-quark decays as a function of $|y^\mu|$ for two different p_T^μ cuts as measured by D0 [26] compared to the prediction of* CASCADE *.*

parton showers, heavy quark excitation in form of the processes $Qg \to Qg$ and $Qq \to Qq$ contributes significantly to the $b\bar{b}$ cross section in the central rapidity region. The $b\bar{b}$ measurements at the TEVATRON can be reasonably well described [29] by the price of including heavy quark excitation and a heavy quark component in the structure function of the proton. However k_t-factorization is superior to the collinear approach with heavy quark excitation, since in the k_t-factorization approach heavy quarks are produced perturbatively only via hard scattering matrix elements, without further additional assumptions.

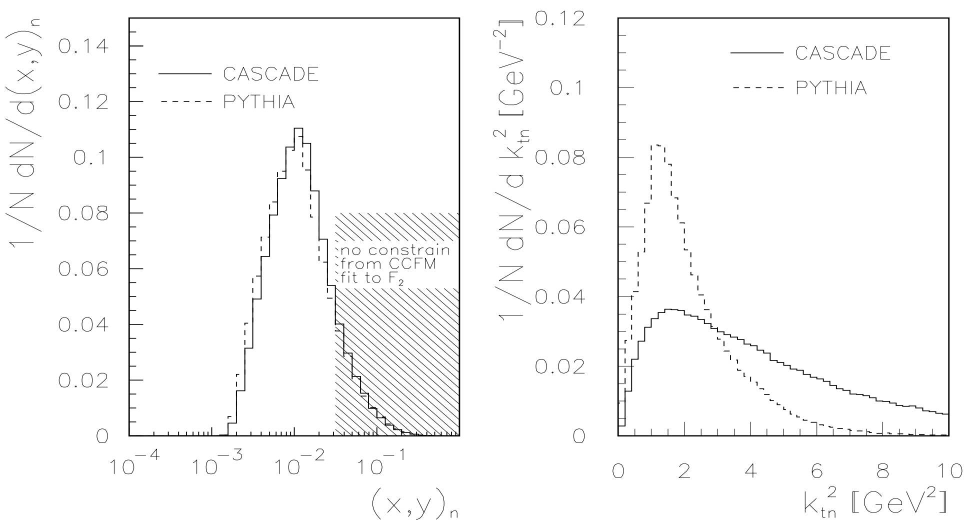

Figure 8: *Comparison of x_n and k_{tn} distributions of the gluons entering the process $g+g \to b+\bar{b}$. Shown are the predictions from* Cascade *representing k_t-factorization with the CCFM unintegrated gluon density and also from* Pythia *representing the collinear approach supplemented with initial and final state DGLAP parton showers to cover the phase space of p_T ordered QCD cascades.*

In Fig. 8 the x_n and k_{tn} distributions of the gluons entering the hard scattering process $g + g \to b + \bar{b}$ (see Fig. 1) are shown and the predictions from the k_t - factorization approach (Cascade) are compared to the standard collinear approach (here Pythia [30] with LO $gg \to b\bar{b}$ matrix elements supplemented with DGLAP parton showers to simulate higher order effects). Whereas the x_n distributions agree reasonably well, a significant difference is observed in the k_{tn} distribution. However this is not surprising: the k_t factorization approach includes a large part of the fixed order NLO corrections (in collinear factorization). Such corrections are $gg \to Q\bar{Q}g$, where the final state gluon can have any kinematically allowed transverse momentum, which could be regarded as a first step toward a non-p_T ordered QCD cascade. The importance of higher order QCD radiation can be estimated from the number of hard gluon radiation N_g with a transverse momentum of the gluon $p_T^g > p_T^{b\bar{b}}$ by defining the probability P_h:

$$P_h(N_g) = \frac{1}{N}\frac{dN}{dN_g}\bigg|_{p_T^g > p_T^{b\bar{b}}} . \tag{12}$$

In a leading order calculation in the collinear approach, even supplemented with DGLAP parton showers, it is expected that $P_h(N_g \geq 1) \to 0$, because of the approximate transverse ordering constraint in DGLAP. In a fixed order α_s^2 (NLO) calculation, $P_h(N_g = 1) \neq 0$, but

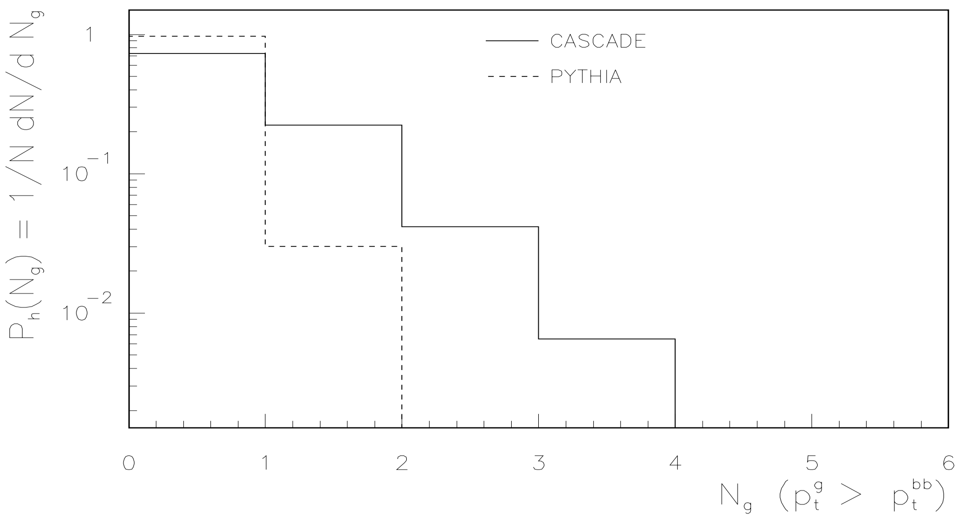

Figure 9: *Probability* $P_h(N_g) = \frac{1}{N}\frac{dN}{dN_g}$ *of hard gluon radiation with* $p_t^g > p_T^{b\bar{b}}$ *in* $b\bar{b}$ *production. Shown are calculations in the* k_t *- factorization approach (*Cascade*) and from the LO DGLAP approach (*Pythia*).*

$P_h(N_g \geq 2) = 0$. In Fig. 9 the probability P_h is shown as a function of N_g for the production of $b\bar{b}$ events with $p_T^b > 5$ GeV, $|y^b| < 1$ obtained in the k_t - factorization approach via Cascade. Also shown for comparison is collinear approach in LO supplemented with DGLAP parton showers obtained from Pythia. In the k_t-factorization approach a significant contribution of hard higher order QCD radiation with $p_T^g > p_T^{b\bar{b}}$ is obtained, which is larger by an order of magnitude compared to the prediction of Pythia.

4 $b\bar{b}$ production at HERA

In [19] the prediction of Cascade for the total $b\bar{b}$ cross section was compared to the extrapolated measurements of the H1 [8] and ZEUS [9] experiments at HERA. Since Cascade generates full hadron level events, a direct comparison with measurements can be done, before extrapolating the measurement over the full phase space to the total $b\bar{b}$ cross section. ZEUS [9] has measured the dijet cross section which can be attributed to bottom production by demanding an electron inside one of the jets. In the kinematic range of $Q^2 < 1$ GeV^2, $0.2 < y < 0.8$, at least two jets with $E_T^{jet1(2)} > 7(6)$ GeV and $|\eta^{jet}| < 2.4$ and a prompt electron with $p_T^{e^-} > 1.6$ GeV and $|\eta^{e^-}| < 1.1$, ZEUS [9] quotes the cross section:

$$\sigma^{b\rightarrow e^-}_{e^+p\rightarrow e^+ + dijet + e^- + X} = 24.9 \pm 6.4^{+4.2}_{-7.3} \text{ pb} \qquad \text{(ZEUS [9])} \tag{13}$$

with the statistical (first) and systematic (second) error given. Within the same kinematic region and applying the same jet algorithm Cascade predicts:

$$\sigma^{b\rightarrow e^-}_{e^+p\rightarrow e^+ + dijet + e^- + X} = 20.3^{+1.6}_{-1.9} \text{ pb}, \tag{14}$$

where the error reflects the variation of $m_b = 4.75 \mp 0.25$ GeV. This value agrees with the measurement within the statistical error. It has been shown in [31], that the extrapolation to the $b\bar{b}$ cross section cross section includes large model uncertainties and the comparison of extrapolated cross sections with predictions from NLO calculations are questionable.

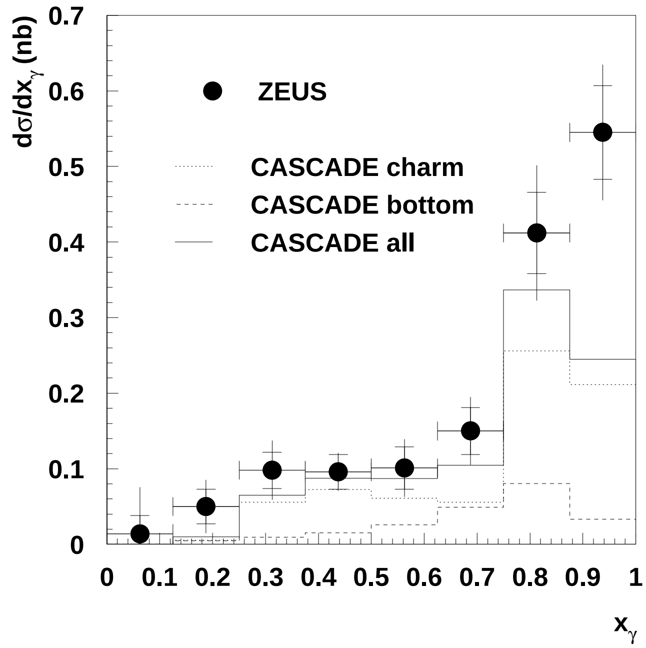

Figure 10: *The differential cross section $d\sigma/dx_\gamma^{obs}$ for heavy quark decays as measured by ZEUS [9] and compared to* CASCADE. *Shown are the charm, bottom and the sum of both contributions to the cross section (Note there is no additional K factor applied).*

It is interesting to note, that the CASCADE result agrees well with the PYTHIA result for the di-jet plus electron cross section, if heavy quark excitation is included in PYTHIA. As in the case of $b\bar{b}$ production at the TEVATRON higher order QCD effects are important, which are already included in the k_t - factorization approach.

In Fig. 10 the differential cross section for heavy quark decays (charm and bottom) as a function of x_γ predicted by CASCADE is compared with the measurement of ZEUS [9]. A significant fraction of the cross section has $x_\gamma < 1$, which is similar to the observation in charm photo-production [19]. In LO in the collinear factorization approach this is attributed to resolved photon processes. However, in k_t - factorization, the x_γ distribution is explained naturally because gluons in the initial state need not to be radiated in a p_T ordered region and therefore can give rise to a high p_T jet with transverse momentum larger than that of the heavy quarks.

The prediction of CASCADE has also been compared to the measurement of H1 [8] for electro-production cross section in $Q^2 < 1$ GeV^2, $0.1 < y < 0.8$, $p_\perp^\mu < 2$ GeV and $35^o < \theta^\mu < 130^o$:

$$\sigma(ep \to e' b\bar{b} X \to \mu X') = 0.176 \pm 0.016(stat.)^{+0.026}_{-0.017}(syst.) \text{ nb}$$

This cross section already includes the extrapolation from measured jets to the muon. In the same kinematic range CASCADE predicts:

$$\sigma(ep \to e' b\bar{b}X \to \mu X') = 0.066^{+0.009}_{-0.007}\ \mathrm{nb}.$$

which is a factor ~ 2.6 below the measurement. It has been shown in [31] that the experimental results from H1 and ZEUS are different by a factor ~ 2. It has been checked, that the difference between the experiments is not due to different kinematics.

5 Conclusion

Bottom production at the TEVATRON can be reasonably well described using the k_t- factorization approach with off-shell matrix elements for the hard scattering process. Also the cross section for b quarks decaying semi-leptonically into muons at small angles is reasonably well described, whereas NLO calculations fall below the data by a factor of ~ 4. One essential ingredient for the satisfactory description is the unintegrated gluon density, which was obtained by a CCFM evolution fitted to structure function data at HERA, showing evidence for the universality of the unintegrated gluon density. The comparison with the data was performed with the CASCADE Monte Carlo event generator, which implements k_t-factorization together with the CCFM unintegrated gluon density.

Measurements of bottom production at HERA are also compared to predictions from CASCADE. The visible dijet plus electron cross section attributed to b-production as measured by ZEUS could be reproduced within the statistical error. However, the situation is different with the H1 measurement: the visible muon cross section is already a factor of 2.6 above the prediction from CASCADE. Further measurements, also differential, are desirable to clarify the situation at HERA.

In general the k_t-factorization approach has now proven to be successful in a wide kinematic range. It is worthwhile to note, that the approach presented here, in addition to the description of $b\bar{b}$ production, is also able to describe charm and jet-production at HERA as well as the inclusive structure function F_2. It is the advantage of this approach that contributions to the cross section, which are of NLO or even NNLO nature in the collinear ansatz, are consistently included in k_t-factorization due to the off-shellness of the gluons, which enter into the hard scattering process.

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